"Senior Housing: An Inter-Generational Solution"

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Abstract

Senior Housing: An Inter-Generational Solution

Classic views of senior housing choice have correctly noted that household family size shrinks with age and resources devoted to elderly housing consumption can then be redirected to other areas. The debate is about the best way to undertake this change. The alternatives are numerous: Sell and buy smaller vs. sell and rent smaller are special cases of the rent-or-buy decision. Accumulated home equity may be used for retirement leisure, unexpected expenses associated with health matters or different kinds of elder care. The equity need not be withdrawn or exhausted if various reverse amortization mortgage plans are employed. Moving in with younger family members is often discussed but nearly as often dismissed as unsuitable for a host of reasons, nonetheless it must be included in the mix of options.

This study looks at an intergenerational solution to the housing problem facing the elderly. It describes how a unique combination of family systems, property rights, investment goals and tax benefits can produce optimal solutions for some families. A theoretical model is presented wherein an older family member optimizes health, housing and bequest options while at the same time in the same transaction a younger family member seeks an optimal investment.

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I. Introduction

When we look back on the first third of the 21st Century it is likely we will notice that a large portion of the population, known in those days as "Baby Boomers", spent several decades confronting the question "What to do with Mom and Dad". Roughly half of that time will have involved their own aging parents. The years remaining will be recalled as "payback" during which they were the object of the same question. This study looks at an intergenerational solution to the housing problem facing the elderly. It describes how a unique combination of family systems, property rights, investment goals and tax benefits can produce optimal solutions for some families.

Classic views of this dilemma have correctly noted that household family size shrinks with age and resources devoted to elderly housing consumption can then be redirected to other areas. The debate is about the best way to undertake this change. The alternatives are numerous: Sell and buy smaller vs. sell and rent smaller are special cases of the rent-or-buy decision. Accumulated home equity may be used for retirement leisure, unexpected expenses associated with health matters or different kinds of elder care. The equity need not be completely exhausted if various reverse amortization mortgage plans are employed. Moving in with younger family members is often discussed but nearly as often dismissed as unsuitable for a host of reasons, nonetheless it must be included in the mix of options. Declining health and rising need for care has introduced a new elder housing vocabulary including "assisted living", "independent living" and "elder day care". Lastly, there is the specter of the skilled nursing facility, a housing option that may be prosaically viewed as "God's Waiting Room."

The purpose of this paper is to examine this difficult challenge and propose a solution that fits the needs of two generations. Acknowledging that young people have resources, energy, risk tolerance and different long-term goals, we propose the elder generation joint venture the housing solution with their children in a rational and business-like fashion. The process will not be perfect nor will it fit every family. But it does lengthen the number of choices.

The paper will proceed as follows. A brief literature review follows this introduction. Discussion of senior housing alternatives is in Section III. Section IV describes the theoretical model. Section V presents an example under two different baseline conditions. Section VI explores two intra-family alternatives from the perspective of the retiree. Section VII looks at the same issue from the standpoint of the younger family member. Section VIII introduces actuarial considerations, followed by concluding remarks in Section IX.

II. Literature review

The literature for this subject is found in several areas. Social scientists interested in family systems consider this the issue of aging. Those interested in land use and real estate economics consider it a subset of housing policy. Finance researchers view the area as part of real estate lending. Thus, the literature is spread over a number of academic journals.

The most common and direct form of realizing the value of homeownership late in life is to sell the home, a decision that involves displacement and transaction costs. The general name of this activity is “home equity conversion” (HEC).[1] Pastalan (1983) lists sale-leaseback, sale of remainder interest, shared housings and special purpose loans among the methods of HEC that avoid a sale and may be considered non-standard forms. He finds that shared housing yields the highest remaining monthly income after housing expense. He also questions the value of any HEC method after a certain age due to the short life expectancy.

Venti and Wise, in a series of articles start (1984) by analyzing the housing choice of low income elderly renters then (1989) investigate how equity changes when elderly owners move, finding that equity is not likely to decrease. Finally (1990), they consider how bequeathable wealth is allocated between housing and other assets.

Early papers taking the mortgage approach include Crossman (1984) who includes a desire for family closeness and, asserting that the elderly prefer homeownership to renting, address the need for renovation financing to keep the elderly safely in their homes. Weinrobe (1984) enlarges on financing alternatives to include sale and leaseback arrangements between related persons and proposes how consumer safeguards may be structured. Important in this latter paper is the suggestion that (a) a life tenancy may be useful and (b) the term selection might be related to life expectancy. Weinrobe wisely concludes that these contractual arrangements arise from common sense planning and the sort of market failure that foments government regulation should not occur.

In a formal approach to the Reverse Amortization Mortgage (RAM), DiVenti and Herzog (1990) observe that special constraints on this sort of underwriting include the fact that the elderly live in older housing stock and are less likely to maintain it, leading to a lower probability that it will

[1] A broader use of the term includes refinancing equity out of the home.

appreciate during the tenure of a RAM. Their modeling necessarily relies on assumptions about the termination of any such arrangement. The major reasons include death, disability, hospitalization, other reasons for a change in location. Lacking data on the plethora of reasons, a simplifying assumption is to choose death as the terminating event.

Phillips and Gwen (1992) consider the matter from an actuarial perspective, correctly noting that there are cross-discipline issues at work. A primary result of their work is the computation of a “crossover point” after which the loan balance is in excess of the value of the property and, under typical RAM terms, unrecoverable. This is a risk unique to this instrument and must be factored into the lender’s pricing and portfolio decisions.

The Journal of the American Real Estate and Urban Economics Association devoted its entire Summer, 1994 issue to elderly housing financing. Articles there address housing decision of the elderly (VanderHart) and analysis of data from the American Housing Survey (Merrill, Finkel and Kutty). A number of risk questions are investigated, including interest rate risk (Boehm and Ehrhardt) and prepayment risk (Klein and Sirmans). The most relevant of these articles to the present effort relates to the risk that the borrower will not maintain the property. Miceli and Sirmans present a model with a perverse incentive to reduce maintenance as borrower equity declines. They argue that lenders will limit their risk in this area by restricting their exposure (making fewer loans) or charging a risk premium.

In 1998 the American Real Estate Society produced a monograph containing fourteen articles that focus mostly on the relationship between health care and housing. The specific topics include a primer on key issues (Bengamin and Anikeeff) and terminology (Scribner and Dalkowski); quantifying supply (Anikeeff and Novitzki) and forecasting demand (Edelstein and

Lacayo); and portfolio (DuBrin) and health care REITs and health care stocks (Terris and Myers).

Rasmussen, Megbolugbe and Morgan (1997) widen the discussion of RAMs, suggesting that they may be used to tap wealth in one's home at different times of life for different reasons such as financing the college education of a child or making estate planning bequests. They also observe that middle age equity-rich persons can use a RAM for indirect elder housing by supporting the housing of parents from monthly RAM payments.

Fratantoni (1999) performs an interesting empirical analysis, nicely supporting his theory that the primary motivation for RAMs is to provide cash for "economic shocks" late in life. He suggests that the RAM variant permitting a committed equity line of credit is more desirable than the usual RAM offering a monthly income.

What these authors develop is an argument for the creative mix of property rights, housing equity wealth and life-cycle planning to address a housing problem associated with aging. What is absent from this literature is consideration of the investment yield and tax implications for one generation when it provides a solution for another. This paper fills that gap in the literature. The financial literature to date focused on estimating the demand and managing the risk for a third party RAM lender who offers a product. This paper argues that many obstacles are removed when the parties "go direct" and eliminate the lender as middle man.

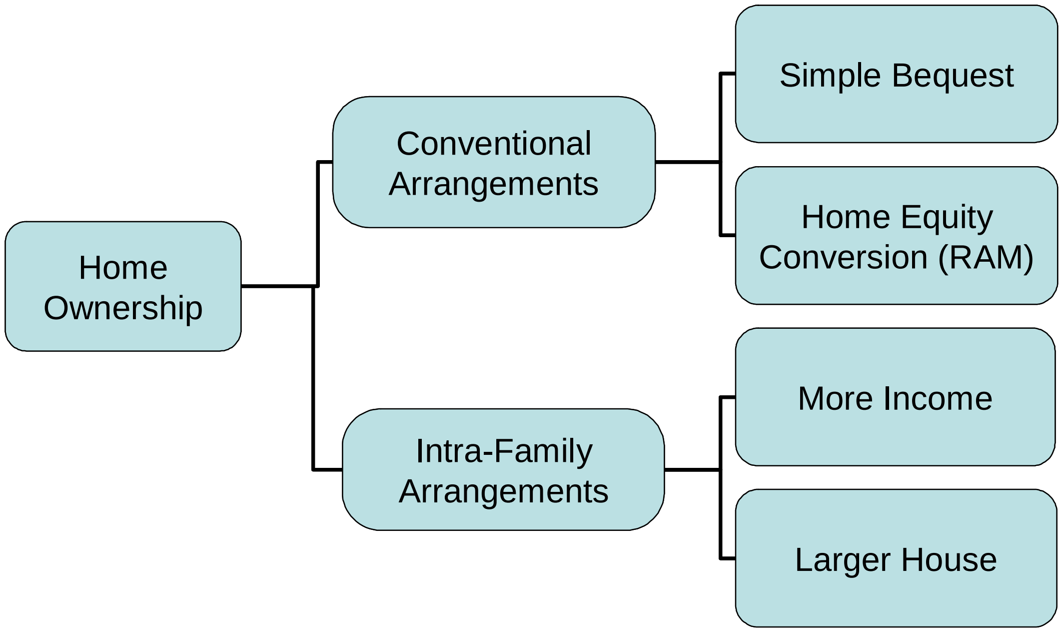
III. Retirement and Creative Financing

One broad definition of creative financing is any specific rearrangement of property rights that simply moves benefits between the parties to a transaction in a unique way. With the increase in retiring baby boomers and a (hoped for) concomitant maturity and success of their progeny, an opportunity exists to consider ways in which one generation can finance the other. For those who have financed their children for many years this version of creative financing may be a pleasant turnabout.

In 1988 Congress authorized a mortgage instrument known as a Home Equity Conversion Mortgage (HECM), more generally known as a Reverse Annuity Mortgage (RAM) because it operates exactly the opposite of the usual loan arrangement. Under an HECM seniors can turn a portion of their home equity into an annuity to overcome a "house-rich, cash-poor" condition. Rather than receiving a lump sum and making payments back to the lender for a number of years, the borrower under an HECM signs all the loan documents but receives no large lump sum initially. Instead, monthly payments are sent to the borrower by the lender for a period of time. At the end, usually when the house is sold following the departure or death of the borrower, the lender gets a lump sum re-payment that includes accrued interest. To build a foundation we will briefly review some of the mechanics of the HECM, but the main idea of this paper is to bypass the institution and structure an intra-family investment that provides economic benefits to two family members in the same transaction. Figure 1 displays a series of binary choices that constitutes the structure of the illustrations that follow.[2]

[2] It is important to recognize that no attempt is made here to solve a general equilibrium problem for senior housing. A number of issues make such a task intractable, not the least of which is the large number of variables involved. Indeed, tackling a problem of this nature involves enough daunting complexity to discourage the most dedicated

Figure 1



To motivate the discussion, suppose that one contemplating retirement would like to move to a different region where home prices are higher but so is appreciation. A common move of this sort is from the colder climes to the Sunbelt, often motivated by health reasons, personal lifestyle or to be closer to grown children. We will assume our retiree owns a debt-free home of modest value in a Midwestern state. Suppose further that a younger family member, perhaps living in a warmer climate to which the retiree would like to move, is interested in an investment that supports both the retirement and estate planning goals of his senior family member. To achieve these ends we will combine important features of two concepts, a life estate and a zero coupon bond.

One normally thinks of owning property in perpetuity. Thus, one has the right to devise one's property to others at death via a will or trust arrangement. Those who inherit then have a similar

researcher. The series of non-exhaustive simulations provided here represent the best, most practical and perhaps the only approach to a problem of this nature.

right, and so on, in perpetuity. In most developed countries it is legally possible to divide such a perpetuity into two time periods, a life estate and a remainder. In its most common form, a retiree would hold a life estate for the last years of his life. Upon his death, the life estate "falls" to the remainderman who owns the remainder in perpetuity. There are many variations. The life estate may be measured by the life of someone else, the retiree may vacate the house prior to death for a variety of reasons, at death the remainder may be further divided to serve yet another life estate holder. To simplify the termination question we assume that our retiree occupies the house until his death and the remainder falls to a younger family member who is financially involved in the housing solution.[3]

Zero Coupon Bonds are also common. The usual form is a long-term bond generally issued by a government agency. It pays no interest until maturity. The purchase price, naturally, is less than the payment at the end. Important to the usual Zero is that the payoff date is certain. An important distinction in the case we will discuss here is that the payoff date is uncertain.

IV. The Retiree's Housing Dilemma

The broad range of housing choices facing a senior citizen include (a) live in a small house, allocating less retirement income to housing, and leave a bequest; or (b) live in a larger house (or its economic equivalent) for the remainder of his life, exhausting all or parts of its equity via reverse amortization payments or some other arrangement that leaves little or no bequest to heirs.

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[3] But is not the retiree's heir, at least with regard to this property, the reasons for which are discussed later.

The model requires a number of terms:

le	=	number of years the retiree will occupy his last house represented by his life expectancy
MaxV	=	the largest value at purchase of the last house to be occupied, a proxy for size
oc	=	operating cost of the house (taxes, insurance, maintenance), a function of the size of the house
b	=	bequest at death
g	=	growth in value over time
inc	=	monthly income of the retiree, a constraint on the size of the loan available and therefore on the value (size) of a purchased house that can be financed
pti	=	payment-to-income ratio, a maximum imposed by a lender
dp	=	down payment on the retirement home, sometimes assumed to be equal to the value of the prior residence
loan	=	loan acquired for purchase or refinance of the house
i	=	interest rate on any mortgage
t	=	total amortization period (in months) of any mortgage obtained
pv_i	=	present value of any income stream obtained or foregone
pv_b	=	present value of a future bequest
d_i	=	discount rate applied to foregone income
d_b	=	discount rate applied to bequest

Before examining the intra-family options we will consider two conventional alternatives. One is the simple ownership of the property with a loan on it; the other is the reverse annuity mortgage. The situations differ substantially from each other in that the simple ownership illustration employs a fact situation that may constrain the retiree to a smaller house. On the other hand, the reverse annuity mortgage usually involves a large home already owned for some time that now has little or no debt. Thus, these alternatives are suggestive of two different retirees.

For the reverse amortization mortgage we also define:

val	=	value of a house to be encumbered with a reverse amortization mortgage
pmt	=	payment due from lender to borrower under a reverse amortization mortgage
ltv	=	lender's maximum loan-to-value ratio under a reverse amortization mortgage

Note that MaxV and val are not the same. The former is a calculated number in one scenario; the latter is an input value under different conditions.

V. Two Conventional Arrangements

As foundation, we begin with two conventional arrangements, the simple bequest of a last residence and the use of a reverse amortization mortgage. To facilitate the exposition, Table 1 provides a set of fixed inputs to serve as an example.

Table 1

Symbol	Value	Notes
dp	135,000	Value of former home used as down payment on new home
g	4%	Annual growth rate on new home
i	6%	Annual interest rate on loan against new home
t	360	Term, in months, of loan against new home
le	6	Life Expectancy in years
oc	2%	Operating costs of new home expressed as a percentage of value
inc	3750	Monthly income for retirement
pti	40%	Lender imposed payment to income ratio
val	300000	Value of new home
ltv	60%	Lender imposed maximum loan to value ratio on Reverse Mortgage
pmt	1500	Monthly payment available for housing (inc * pti)
d_i	10%	Discount rate applied to foregone income
d_b	18%	Discount rate applied to bequest

a. The simple bequest

In the first illustration one sells his existing debt free home of modest size and value and uses the proceeds as a down payment for the purchase of a new house in a preferred location. He lives in this house until he dies and leaves the house to his heirs.

Our retiree may borrow an amount, loan, at interest rate, i, that a portion of his income, inc, represented by a lender imposed maximum payment-to-income ratio, pti, can repay over a full amortization period, t. In the interest of simplicity, we ignore other homeownership operating costs at this stage.

The maximum value of the house he can purchase, maxV, is then equal to the amount he can borrow, plus the value of his old residence used as a down payment, dp.[4]

$$\max \boldsymbol{V} = \boldsymbol{dp} + \frac{\left(1 - \left(\frac{1}{\left(1+\boldsymbol{i}\right)^{\boldsymbol{t}}}\right)\right)}{\boldsymbol{i}} \boldsymbol{inc}\ \boldsymbol{pti} \tag{1}$$

For the example in Table 1 the maximum size house our retiree can purchase is $385,187.[5]

The sale price, s, at death is the value, maxV, increased by growth, g, compounded over the life expectancy, le, which for our example is $487,385.

$$\boldsymbol{s} = \max \boldsymbol{V} \left(1+\boldsymbol{g}\right)^{\boldsymbol{le}} \tag{2}$$

[4] The ratio in the middle of Equation (1) is just the Ellwood equation for mortgage balance. Throughout this paper we use monthly income and assume mortgage payments are made monthly. Thus we will occasionally have to introduce a factor of 12 when referring to annual returns.
[5] We ignore for simplicity the fact that the lender may base its underwriting on the size of the total monthly ownership cost including principal, interest, property taxes and insurance.

The bequest, b, is then merely the equity remaining at life expectancy, le, which is the difference between the value and the loan balance at sale.

$$\boldsymbol{b} = \frac{\left(\left(1+\boldsymbol{i}\right)^{12\boldsymbol{le}-\boldsymbol{t}}-1\right)\boldsymbol{inc\,pti}}{\boldsymbol{i}} + \left(1+\boldsymbol{g}\right)^{\boldsymbol{le}}\left(\boldsymbol{dp} + \frac{\left(1-\frac{1}{\left(1+\boldsymbol{i}\right)^{\boldsymbol{t}}}\right)\boldsymbol{inc\,pti}}{\boldsymbol{i}}\right) \tag{3}$$

Inserting appropriate Table 1 values into Equation (3) heirs may expect to receive a bequest of $258,719 in six years.

b. The Reverse Amortization Mortgage

In the second conventional alternative we consider a different retiree who owns a larger house free of debt and wishes to generate monthly income from his home equity without selling and moving. The RAM lender grants the loan based on the borrower’s life expectancy, le, the value of the house, val, interest rate, i, and payment amount, pmt. The lender sets a maximum the loan can grow to based on the loan to value ratio, ltv.

Two issues are of interest in this case. First, the lender attempts to set the loan maximum so as not to exceed the value of the house. This has implications for the second issue as it determines the number of payments the retiree receives. The balance is between an unacceptable risk for the lender or an inconvenient cessation of income for the retiree.

The monthly payment is a function of the value of the home, the interest rate, the loan to value ratio and the growth in value, per Equation (4).

$$\boldsymbol{income\,payment} = \frac{\boldsymbol{i}}{\left(1+\boldsymbol{i}\right)^{12\boldsymbol{n}}-1}\boldsymbol{ltv\,val}\left(1+\boldsymbol{g}\right)^{\boldsymbol{n}} \tag{4}$$

Table 2 reflects how payment amount and the number of payments change with changes in the growth assumption and permitted maximum LTV.

Table 2

	Growth	LTV	LE	Income	Max Pmts
Growth + High LTV	4%	60%	6	$2635.81	130
No Growth + High LTV	0%	60%	6	$2083.12	94
Growth + Low LTV	4%	40%	6	$1757.21	85

Taking n in Equation (4) as life expectancy and graphing income payment against time in Figure 2 we see that the more permissive values from Table 1 which allow a relatively high loan to value ratio against a growing value always produce a higher income. Because the outcome is tilted in favor of the retiree when high loan to value ratios are allowed against a growing value, lenders are understandably reluctant to enter into loan agreements of that nature. But when the tradeoff is between allowing a growth assumption and a lower LTV, Figure 2 shows that, for the example values, the decision changes when life expectancy exceeds ten years.

Figure 2

Figure 2 goes here

The purpose behind the exercise in this section is not to generalize the idea of reverse amortization mortgages or advocate for any particular underwriting position. Rather we expose the variables that go into the decision for both parties as these variables influence the discussion of intra-family solutions, to which we now turn.

VI. Intra-Family Alternatives – Retiree's Position

Economics only partially control intra-family situations. This is appropriate and can become a benefit. To facilitate this approach we shall assume the reverse annuity mortgage option is not available because the retiree does not own a home of sufficient size to produce the desired results. There are two ways to develop such a financing scheme.

Should someone be willing to purchase a house for our retiree to live in for his lifetime but with no right to devise by will, the retiree would have additional discretionary income. This, which we will call the More Income Viewpoint, considerably enhances the retiree's retirement lifestyle.

Alternatively, the retiree could live in a house he could not otherwise afford if constrained by the loan qualifying payment-to-income ratio. We will call this the Larger House Viewpoint. This variation is just a special case of lifestyle enhancement in which the larger residence is how one elects to apply larger disposable income arising from the intra-family arrangement.

a. The Income Viewpoint

In the first conventional approach of Section V, our retiree essentially "purchases" the satisfaction of leaving a bequest by incurring the obligation to make loan payments and forgoing the benefits associated with more discretionary income he would have had during his lifetime if he did not have loan payments to make. The income viewpoint, the first of the two “intra-family” approaches amounts to "selling" that satisfaction in return for the enhanced present income. The interesting question is: how much of one is the other worth?

The trade off is between leaving a bequest, b, and income, inc. In economic terms, a rational retiree maximizes his utility by choosing the greater of these two. Such a calculation involves mortality assumptions that can, at times be difficult or uncomfortable to make.

Earlier we reached a value of $258,719 for the bequest. To make a fair comparison we need to know the present value of the income foregone in order that a bequest may be left. If our retiree is able to live in a house without paying loan payments he enjoys that income for the remainder of his life. The present value of this income, Equation (5), is dependent on the discount rate, d_i , chosen.

$$\boldsymbol{pv_i} = \frac{\left(1 - \frac{1}{(1+\boldsymbol{d_i})^{12\boldsymbol{le}}}\right)}{\boldsymbol{d_i}}\left(\boldsymbol{inc\ pti} - \frac{\boldsymbol{oc\,val}}{12}\right) \qquad (5)$$

Using Table 1 values we have a present value of the foregone income of $53,978.70. As the bequest is larger, a rational person would choose to buy a house, make payments on a loan and leave a bequest.

Present value may imperfectly adjust for the difference between the value our retiree places on his own consumption and the value he places on financing the future consumption of others.

Discounting the two at the same rate seems improper. It seems at least reasonable that the discount rate for dollars one will consume should be less so that those dollars are valued more. Using data from Table 1, where $d_i < d_b$, the present value of the bequest becomes $88,567. As this is still more than the present value of the foregone income, our retiree still buys a house and leaves a bequest. As the discount rate applied to the bequest rises, eventually the retiree opts to have someone else buy him a house, someone who will receive the house at his death.[6]

So for the income viewpoint, the decision turns on how the retiree values dollars he may consume vs. how he values dollars he leaves behind. In Figure 3 we see that, given data from Table 1, the present value of the bequest dominates until the discount rate exceeds 27%.[7]

Figure 3

Figure 3 goes here

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b. The Larger House Viewpoint

An institutional lender evaluates risk based on the probability of repayment taking place over the investor's lifetime. As there is a cap on his dollar return (all interest paid or accrued plus the principal), the lender makes a loan governed by the realities of (a) the income the retiree has during his lifetime to make payments and/or, (b) the liquidation value of the property needed to retire any balance remaining at the retiree's death. The remainderman as "lender" has a different

[6] We assume that the "someone" who buys the retiree a house is *not* the heir who would otherwise receive the house at his death. If this were not the case the retiree would be, in a sense, merely deciding the form of the bequest

[7] One could argue that the decision turns on the difference between the two discount rates. This becomes an interesting simulation problem, but one that distracts us from our story.

perspective. Since he captures the entire (uncertain) value of the property at death, the remainderman's payoff prospects are different. It is also possible that an older relative's care of a larger property for the remainderman can produce positive results for the remainderman not included in these computations.

Let us begin by noting how the retiree will approach the possibility of a larger house. Remember that "larger" is just a metaphor for "better" in some tangible way. The house may be better located, newer, have a better view or otherwise in some sense be more desirable than the house the retiree might purchase. Or it might be larger. We assume that all of these desirable attributes will be captured in a higher price making easier the *measurement* of "larger" or "better".

Suppose that the retiree's self imposed limit on the portion of his income he will spend on housing is half of what a lender will allow. That is, he wishes to have the most house he can support, paying in operating costs, oc, half the amount as his loan payment would have been had he purchased the property. The point is that our retiree has a housing budget that is a self-imposed constraint on the size of house he is willing to support whether that support is in the form of loan payments or upkeep.[8] Clearly, either "bigger" or "better" is more feasible without loan payments. We will suppose that annual operating costs on an expensive residence run 2% of its purchase price. Thus, he can "carry" a house the value of which is equal to the ratio of his annual housing budget to operating costs. This value is a function of his disposable income, his housing budget constraint and the operating cost of the house as shown in Equation (6).

$$\boldsymbol{LgeHseVal} = \frac{12\boldsymbol{inc\ pti}}{.5\boldsymbol{oc}} \tag{6}$$

[8] There is some discontinuity in the examples here as one must know the value of the house to determine the amount of the operating costs. For the more income perspective we used the fixed value, val, in Table 1 for this number. Here we use MaxV. There is no good solution for this as each case will stand on its own.

For Table 1 data this means our retiree will live in a \$450,000 house. If we assume, naively, that the utility of different houses is represented by the difference in their values, the retiree chooses the greater of (a) the difference between the value of the house he could purchase independent of any life estate arrangement and the value of the house he could acquire using the life estate technique or (b) the bequest, again using appropriate discounts. The difference between the house he could buy, maxV, of \$385,187 and the larger house, LgeHseVal, of \$450,000 is \$64,813. Compared to the \$88,567 present value of the bequest our retiree still leaves a bequest.

Setting the difference between the two houses equal to the equation representing the present value of the bequest and isolating the pti, Equation (7), we solve for an indifference point based on the portion of the retiree’s income he is willing to devote to housing. For our Table 1 example we find that, if all else is equal and the retiree is willing to use 45.6% of his income for housing rather than the 40% the lender would allow, he is indifferent between the large house and the bequest.

$$\boldsymbol{pti} = \frac{\boldsymbol{dp}\left(\left(\boldsymbol{g}+1\right)^{\boldsymbol{le}}+\left(\boldsymbol{d}_{b}+1\right)^{12\boldsymbol{le}}\right)\boldsymbol{i}\left(\boldsymbol{i}+1\right)^{t}\boldsymbol{oc}}{\boldsymbol{inc}\left(12\boldsymbol{i}\left(\boldsymbol{i}+1\right)^{t}\left(\boldsymbol{d}_{b}+1\right)^{12\boldsymbol{le}}+\left(\left(\boldsymbol{g}+1\right)^{\boldsymbol{le}}+\left(\boldsymbol{d}_{b}+1\right)^{12\boldsymbol{le}}-\left(\boldsymbol{i}+1\right)^{\boldsymbol{le}}-\left(\left(\boldsymbol{g}+1\right)^{\boldsymbol{le}}+\left(\boldsymbol{d}_{b}+1\right)^{12\boldsymbol{le}}-1\right)\left(\boldsymbol{i}+1\right)^{t}\right)\boldsymbol{oc}\right)} \quad (7)$$

The qualifier "if all else is equal" is important. Combining the variables using different values provides an infinite number of permutations. For instance, using 30% for the discount rate produces a pti indifference point of 35.4%, re-emphasizing the importance of the discount rate selection.

VII. Intra-Family Alternatives – The Remainderman’s Position

The Remainderman's position is conceptually much simpler. He may be viewed as buying a zero coupon bond with an uncertain payoff date and amount. We assume that the Remainderman buys the house, maxV, and concurrently sells a life estate to the retiree for the amount the retiree realizes from the sale of his old residence, dp.[9] In that way the Remainderman really is providing financing, creative or not, for he takes the place of the lender. His investment is the amount of the loan he supplies.[10] The payoff is sale price of the property, an unknown amount, at the death of the retiree, an unknown date.

a. The More Income Case

Equation (8) represents the annual return, r_i , to the remainderman for the Income Case

$$\boldsymbol{r_i} = \frac{\boldsymbol{Log}\left[\dfrac{\boldsymbol{s}}{\boldsymbol{loan}}\right]}{\boldsymbol{le}} \tag{8}$$

The second term on the right hand side of Equation (1) represents the equation for the loan. Using Table 1 data that amount becomes $250,187. Using Equation (2) the sale price is $487,385 so at Table 1 life expectancy the remainderman can expect to earn a return of 11.11% per annum.

Not surprisingly, Figure 4 shows that the return is negatively related to life expectancy and positively related to appreciation but to a much smaller degree. The choice of relative to stand in as lender is critical. One does not want to create a perverse incentive in such an arrangement.

[9] The choice of the size of the house to buy is highly variable. To illustrate this we have chosen maxV for the more income case and size of the house that can be supported by the retiree’s budget for operating costs for the large house case.

[10] One must not overlook the complicating but important reality that the remainderman may choose to finance his entire contribution in the form of a conventional loan. Under these circumstances the remainderman is “lending his credit” by qualifying for a loan the retiree could not obtain. This has the further desirable effect of putting the remainderman’s income dollars to work rather than tying up a large amount of capital.

Measuring the utility our remainderman gains from his relations' longevity (or lack of it!) is at best an unsavory task.

Figure 4

Figure 4 goes here

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b. The Large House Case

The larger house alternative in Equation (9) may be less attractive for the younger party. One reason is that in our example the retiree's purchase price for the life estate is limited to the value of his former residence. So even though the growth takes place on a bigger number, unless the larger house comes with a larger growth rate, because of the larger investment represented by the remainderman’s loan this alternative yields less.

$$r_{LgeHse} = \frac{Log\left[\frac{LgeHseVal\left(1+g\right)^{le}}{LgeHseVal-dp}\right]}{le} \qquad (9)$$

For Table 1 data this alternative yields slightly less than 10% per annum. The return is again negatively related to life expectancy but the decline is not quite so precipitous for Table 1 data. If the larger house comes with higher growth, the return is respectable across the likely range of the investment time horizon (see Figure 5).

Figure 5

Figure 5 goes here

VIII. The Use of Mortality Tables

Actuarial science is a branch of statistics that joins concepts from probability, risk theory, business, and economics. It is the basis for pricing insurance products and its insight applies to products that are not actually insurance but have insurance-like characteristics. Actuarial concepts can be employed for our intra-family financing problem. It can more accurately predict future outcomes and improve investment decisions, as the outcome is in part determined by mortality.

We have been rather cavalier about providing fixed values in Table 1 where life expectancy is a variable of interest. Up until now we have left to the reader's subjective estimate the task of predicting the term of the life estate arrangement. While this shall remain both subjective and speculative in practice, using mortality tables at least offers some guidance.[11]

One must be cautioned about using the mortality tables for individuals. Mortality tables are based on a large pool of people and report the portion of the people in that pool that can be expected to die during or survive until the end of any one year. For individuals the "expectation"

[11] A variety of different options and illustrations in an interactive setting relying on user input is planned for www.mathestate.com. The prototype analysis tool is in development and may be previewed at http://www.hostsrv.com/webmac/app1/MSPScripts/webm1016/Remainderman.jsp until the tool is added to the list of interactive models presently available at www.mathestate.com.

is far less precise, variance from expectation can be considerable and dependent on a host of personal factors that may or may not be representative of actuarial results in a large pool.[12]

Nonetheless, conditional probability is very powerful. Rather than just assume a fixed life expectancy we will introduce four common and familiar variables on which to condition our expectation about the length of the life estate arrangement. These are age, gender, smoker (Y/N), and general health (0 = average; and values less and more than 0 are, respectively, below and above average).

Actuarial mathematics extends the concept of present values to actuarial present values (APV). Until now, to find PVs, we discounted the future cash flows with interest. The actuarial present value discounts with interest and mortality. Suppose, for example, that at age *x*, you purchase a one-year term insurance policy. The insurance company will pay your beneficiary a benefit of $1000 at the end of the year, if you die within the year. The APV of the insurance benefit is shown in Equation (10).

$$\boldsymbol{APV} = \frac{1000}{1+\boldsymbol{i}}\boldsymbol{q}_{\boldsymbol{x}} \tag{10}$$

where q_x is the probability that a person aged *x* will die within the next year. That probability is called a mortality rate.

To calculate his return, the remainderman is interested in the APV of the residual when the life estate falls. Thus, we can combine the information from Table 1 data with information about the person by whose life the life estate is measured and approximate a variety of expectations.

[12] Another important aspect of this reality is that institutional lenders are able to spread risk across a similar pool, something not available to a family member and a single retiree.

Using the appropriate mortality rates, for a retiree aged *x*, we can replace the present value of foregone income arrived at earlier using Equation (5) by its actuarial present value in Equation (11).

$$E\left(pv_i\right)=\sum_{k=1}^{\infty} pv_i\left(k\right)_{k-1}p_x q_{x+k} \tag{11}$$

where: $pv_i(k)$ is the present value of foregone income evaluated at k = le and ${}_{k-1}p_x q_{x+k}$ is the probability that the retiree survives k-1 years, and then dies during the k^{th} year.

Similarly, using Equation (12), we can find the actuarial present value of the bequest for a retiree aged x.

$$E\left(pv_b\right)=\sum_{k=1}^{\infty}\frac{b\left(k\right)}{\left(1+d_b\right)^k}\,{}_{k-1}p_x q_{x+k} \tag{12}$$

where: b(k) is the present value of the bequest, evaluated at k = le and ${}_{k-1}p_x q_{x+k}$ is the probability that the retiree survives k-1 years, then dies during the k^{th} year.

Other values, such as the expected return and those values associated with the large house case, are similarly discounted using mortality rates in addition to the time value discount.

Expanding Table 1 data we will add mortality data for our retiree to reflect that she is a 75 year-old non-smoker in average health. Thus, her life expectancy is 12.64 years. Table 3 shows the results of our new functions with the new mortality information.

Table 3

Expected Present Value of Bequest	$50,597.10

Expected Present Value of Foregone Income	$108,995.00
Expected Return (Income Case)	9.56%
Expected Return (Large House Case)	8.58%

Using the same inputs and focusing solely on how mortality affects returns, in Figure 6 we see the expected increase in return with age.[13] However, we also see that returns for the male non-smoker are less than female smokers only in earlier years. As the participants age mortality for the male non-smoker exceeds that for the female smoker, resulting in a rise in return for these participants.

Figure 6

Figure 6 goes here

A final comment about mortality tables is in order. From time to time actuaries revise these tables to reflect actual experience. People simply live longer now than they did 25 years ago with the result that life estate arrangements such as those suggested here show lower returns. The mortality table we have used to this point is the latest, the Commissioner's Standard Ordinary Mortality Table for 2001 (CSO2001). Until just a few years ago the equivalent table was for the year 1980 (CSO1980). A comparison in Table 4 indicates how the calculations differ for our example depending on the mortality table used.

Table 4

	CSO 2001	CSO 1980

[13] The notion of APV can be further extended to the retiree's perspective as well, something left to the reader as an exercise.

Expected Present Value of Bequest	$50,597.10	62182.50
Expected Present Value of Foregone Income	$108,995.00	98651.40
Expected Return (Income Case)	9.56%	10.94%
Expected Return (Large House Case)	8.58%	9.72%

Repeating the plot of four categories over ages 55-90 using the 1980 CSO mortality table in Figure 7, we see that the 2001 revision not only shows lower returns at all ages (the expected outcome when longevity increases) but widens the difference in the classes.

Figure 7

Figure 7 goes here

IX. Conclusion

This analysis could stand for the reason many seniors rent. The complexities of this paper are bewildering enough to anyone not dealing with the challenges of aging. There are even more alternatives that approach the task differently. A shared appreciation mortgage or simple joint tenancy, are just two other possibilities that can achieve similar goals. The important general point is that the United States has an economic system capable of precisely describing a large variety of property rights that can be combined in very specific ways. A talented estate planning attorney and a careful real estate analyst can craft an ownership arrangement tailored to individual needs.

It should be noted as well that the ideas presented here work best in low interest rate environments. Interest rates are not sensitive to mortality. Thus, relatively low returns modeled here may appear attractive when alternate yields are below them. But as interest rates rise and mortality dependent yields remain relatively low, the economic drivers of this idea must give way to non-economic factors.

Throughout this entire paper we have deliberately ignored taxes. This should not be done when a transaction of this type is contemplated. The ordinary income tax questions include who takes the deduction for paying property taxes. There is a property tax/valuation question in states that re-assess on transfer of title. Estate tax questions hinge on the size of the estate, the size of the exemption and other factors. Finally, the capital gains taxes must not be ignored. When the life estate falls the Remainderman can move into the property for a short time establishing it as his primary residence and then, under present US tax law, sell it with no tax due on gains up to $500,000 ($250,000 if filing single). These are powerful benefits and costs that should be included in the analysis.

Due to personal considerations, there are usually non-economic issues at work here. Hopefully these are positive. Numerous family benefits may be realized when older relations are close by (although opposite results can occur). It is assumed that this sort of transaction only takes place among stable, harmonious relations. If so, benefits not measured in dollars could enhance the financial decision in ways not available via conventional lending arrangements. Nonetheless, if the transaction is framed in economics offering a baseline of reasonable financial merit, family members can proceed in a way that minimizes the possibility of one becoming the dependent of the other.

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