2nd Annual Acadia University Undergraduate Mathematics Competition
Time Limit: 2hrs

Instructions: More credit will be given for complete, fully justified solutions. No calculators or other aids are allowed. Complete your rough work on the scrap paper provided and write your final solutions in the exam booklet. Only the exam booklet is to be submitted. Have fun!!!

The next three problems are each worth 10 points

---

1. The median of five positive integers is 15, the mode is 6 and the mean is 12. What are the five numbers?

2. A child on a pogo stick jumps 1 foot on the first jump, 2 feet on the second jump, 4 feet on the third jump, and so on, in general 2^{n-1} feet on the nth jump. Can the child get back to the starting point by a judicious choice of directions?

3. Find a polynomial $P(x)$ so that $P(x)$ is divisible by $x^2 + 1$ and $P(x) + 1$ is divisible by $x^3 + x^2 + 1$

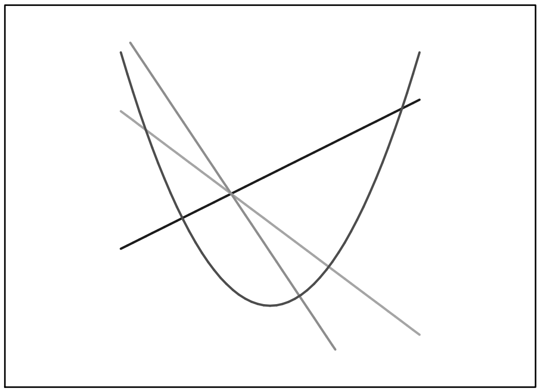
The next four problems are each worth 15 points

---

4. Consider a six-sided die which is to be labelled with the numbers $1-6$. We must have 1 and 6 on opposite sides, 2 and 5 on opposite sides and 3 and 4 on opposite sides. Consider two ways of labelling the die as equivalent if you can rotate the die to make the first labelling match the second. In how many nonequivalent ways can the die be labelled?

5. Prove that at any party, there are two people who have the same number of friends present at the party.

6. Let M be a 3×3 matrix with all entries drawn randomly (and with equal probability) from $\{0, 1\}$. What is the probability that $det(M)$ will be odd?



7. Suppose $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) are three points on the parabola $ay = x^2$ which have the property that their normal lines intersect in a common point. Prove that $y_1 + y_2 + y_3 = 0$.

The next three problems are each worth 20 points

---

8. Sum the infinite series

$$\frac{3}{1 \cdot 2 \cdot 3} + \frac{5}{2 \cdot 3 \cdot 4} + \frac{7}{3 \cdot 4 \cdot 5} + \cdots$$

9. Suppose it started to snow one morning at a constant rate. At 8:00 AM a snowplow leaves the garage and by 9:00 AM is had gone 2 miles. By 10:00 am it had gone 3 miles. Assuming the snowplow removes a constant volume of snow per hour, show that it had started to snow $(\sqrt{5} - 1)/2$ hours ago (or 37 min, 5s).

10. Shuffle an ordinary deck of 52 cards. On average, how far from the top will the first Queen be?